CCFU Proof 27

Unique GL(7)-Orbit

Given. $\Omega_W = \tau \wedge \mathrm{ev} + \mathrm{vol}_W + \mathrm{vol}_{W^*}$ [Proof 21].

Step 1 — Orbit uniqueness (algebraic). Ω_W belongs to the 3-parameter family $\Omega_{a,b,c}$. All $abc \neq 0$ are GL(7)-equivalent [Proof 21, Step 5]. Therefore the GL(7)-orbit of Ω_W is unique. ■

Step 2 — No compact alternative. $\mathrm{sig}(b_{\Omega_W}) = (3, 4)$ [Proof 24]. Split, not compact. The unique orbit is split. ■

Step 3 — Consistency with exhaustive enumeration. Verification (CCFU Script 3):

1524 C_2-compatible patterns with $\dim \mathrm{Stab} = 14$.
1523 split, 1 compact.
All 1523 split: Killing $(8, 6)$, ad-commutant $1 \to \mathfrak{g}_2(\mathrm{split})$.
All consistent with the unique split orbit of Ω_W. ■

Step 4 — Direct GL(7) certificates. Verification (CCFU Script 4):

4 different patterns tested against Ω_{old} (7 terms).
Ω_{old} and Ω_W are in the same GL(7)-orbit [Proof 22].
All 4: $h \in \mathrm{GL}(7)$ and $c \neq 0$ found such that

$$h^*\Omega_{\mathrm{old}} \approx c\,\Omega,$$

with residual $\sim 10^{-24}$. Since multiplication by a nonzero scalar is absorbed by a scalar element of GL(7), this still gives a GL(7)-orbit certificate. ■

The orbit is unique. The proof is algebraic (Steps 1–2). The computational verification (Steps 3–4) is additional confirmation, not the proof.

[Dependencies: Proofs 21, 22, 24.]